5d SCFTs, Brane Webs, Geometric Engineering and the Tangram

Ignacio Carreño Bolla

Universidad de Oviedo and ICTEA

Based on:

ICB, S. Franco, D. Rodriguez-Gomez [2411.01510]

Jornadas ICTEA, Oviedo, 19^{th}-20^{th} June 2025

University of Oviedo

Table of Contents

Contents

- **QFTs** are one of the most successful theories coming from theoretical physics (SM and condensed matter)!

Supersymmetry

- **QFTs** are one of the most successful theories coming from theoretical physics (SM and condensed matter)!
- **Supersymmetry** (SUSY) is an important *theoretical laboratory* for the study of QFTs (confinement, many AdS/CFT, dualities...)

$$\text{SUSY:} \quad \text{bosons}\,(\phi) \leftrightarrow \text{fermions}\,(\psi)$$

- As a consequence, bosons and fermions group into **supermultiplets** closed under the SUSY swapping

$$\Phi = (\phi, \psi)$$

One can then write Lagrangians as one does with "regular" QFTs.

5d SQFTs

- Why 5d if our universe is (macroscopically) 4d? Consider Yang-Mills in arbitrary dimensions

$$S_{\mathrm{YM}} = -\frac{1}{4g^2}\int d^d x \ \mathrm{Tr}(F^{\mu\nu}F_{\mu\nu})$$

  the dimension of the coupling is $[g^{-2}]{=}d-4$. If $d=5$, its dimensionful and YM **cannot be renormalized!**

5d SQFTs

- Why 5d if our universe is (macroscopically) 4d? Consider Yang-Mills in arbitrary dimensions
$$S_{\mathrm{YM}} = -\frac{1}{4g^2}\int d^d x\ \mathrm{Tr}(F^{\mu\nu}F_{\mu\nu})$$
the dimension of the coupling is $[g^{-2}]=d-4$. If $d=5$, its dimensionful and YM **cannot be renormalized!**
- The theory is not defined in the UV ($E \to \infty$), so gauge theories (e.g. SM) are only **low energy EFTs** (defined up to an energy scale $\Lambda < \infty$).

5d SQFTs

- Why 5d if our universe is (macroscopically) 4d? Consider Yang-Mills in arbitrary dimensions

$$S_{\text{YM}} = -\frac{1}{4g^2} \int d^d x \ \text{Tr}(F^{\mu\nu} F_{\mu\nu})$$

the dimension of the coupling is $[g^{-2}]{=}d-4$. If $d=5$, its dimensionful and YM **cannot be renormalized!**

- The theory is not defined in the UV ($E \to \infty$), so gauge theories (e.g. SM) are only **low energy EFTs** (defined up to an energy scale $\Lambda < \infty$).
- How can we define gauge theories from the UV in 5d?

UV fixed points

- Using string theory, it was found that there exist **strongly coupled** $(1/g^2 = 0)$ and scale invariant (**conformal**) interacting SUSY theories !

UV fixed points

- Using string theory, it was found that there exist **strongly coupled** ($1/g^2 = 0$) and scale invariant (**conformal**) interacting SUSY theories !
- We can then, "turn on" the YM theory by making $1/g^2 \neq 0$

$$\mathcal{L} = \mathcal{L}_{UV} + 1/g^2 \mathcal{L}_{YM}$$

This is a **mass deformation** of the theory.

UV fixed points

- Using string theory, it was found that there exist **strongly coupled** $(1/g^2 = 0)$ and scale invariant (**conformal**) interacting SUSY theories !
- We can then, "turn on" the YM theory by making $1/g^2 \neq 0$

$$\mathcal{L} = \mathcal{L}_{UV} + 1/g^2 \mathcal{L}_{YM}$$

  This is a **mass deformation** of the theory.
- Since $[1/g^2] = 1$, the deformation is **relevant** and will dominate if we consider the theory at low energy.

UV fixed points

- Using string theory, it was found that there exist **strongly coupled** ($1/g^2 = 0$) and scale invariant (**conformal**) interacting SUSY theories !
- We can then, "turn on" the YM theory by making $1/g^2 \neq 0$

$$\mathcal{L} = \mathcal{L}_{UV} + 1/g^2 \mathcal{L}_{YM}$$

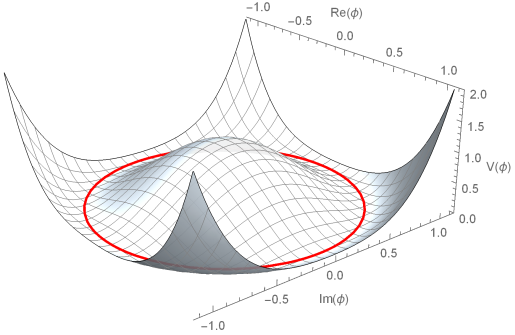
  This is a **mass deformation** of the theory.
- Since $[1/g^2] = 1$, the deformation is **relevant** and will dominate if we consider the theory at low energy.
- All in all,*starting from this* **UV SCFT** *theory we recover gauge theories at low energy!*

Supermultiplets of 5d SCFTs

- 5d SUSY theories have two types of supermultiplets:
  - **Vector:** 1 A_μ gauge field + 2 Majorana fermions ψ_i+ 1 ϕ real scalar.
  - **Hypermultiplet:** 2 complex scalars H_α+ 1 Dirac fermion Ψ.

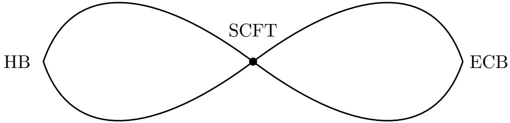
Supermultiplets of 5d SCFTs

- 5d SUSY theories have two types of supermultiplets:
  - **Vector:** 1 A_μ gauge field + 2 Majorana fermions ψ_i+ 1 ϕ real scalar.
  - **Hypermultiplet:** 2 complex scalars H_α+ 1 Dirac fermion Ψ.
- Scalars appear in the action in a superpotential V (polynomial of ϕ). The minima $\partial V/\partial\phi = 0$ defines a **moduli space**.



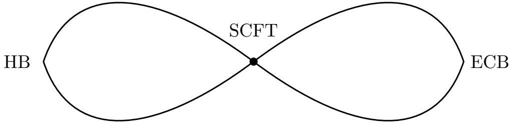
Moduli Space

- The moduli space has as coordinates the (VEVs of) the scalars and the origin is the **SCFT** point



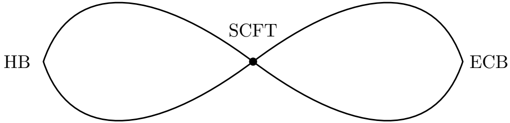
Moduli Space

- The moduli space has as coordinates the (VEVs of) the scalars and the origin is the **SCFT** point
  - **Extended Coulomb branch (ECB)**: ϕ from **vectors**.
    If *gauge*: $G \to U(1)^r$ **(Coulomb branch)**.
    If *global*: **mass deformations**.



Moduli Space

- The moduli space has as coordinates the (VEVs of) the scalars and the origin is the **SCFT** point
  - **Extended Coulomb branch (ECB)**: ϕ from **vectors**.
    If *gauge*: $G \to U(1)^r$ (**Coulomb branch)**.
    If *global*: **mass deformations**.
  - **Higgs Branch (HB)**: H_α hyper. Break completely $G \to 0$.



Contents

Geometric Engineering

- The UV SCFTs are strongly-coupled ($1/g^2 = 0$) and are hard to define. But we can use geometry via **geometric engineering**!

Geometric Engineering

- The UV SCFTs are strongly-coupled ($1/g^2 = 0$) and are hard to define. But we can use geometry via **geometric engineering**!
- Consider **M-theory** (11d) on a geometry $\mathbb{R}^{1,4} \times M_6$. This M_6 is a 6d geometry called a **Calabi-Yau (CY)**, that "is compatible with SUSY".

Geometric Engineering

- The UV SCFTs are strongly-coupled ($1/g^2 = 0$) and are hard to define. But we can use geometry via **geometric engineering**!
- Consider **M-theory** (11d) on a geometry $\mathbb{R}^{1,4} \times M_6$. This M_6 is a 6d geometry called a **Calabi-Yau (CY)**, that "is compatible with SUSY".
- This CY should also be **singular**. These singularities can be removed in two ways:
  - **Resolution:** replace the singular point p by a sphere $S^2 \cong \mathbb{P}^1$.
  - **Deformation:** the CY is defined by polynomial equations

$$f(z) = 0 \rightarrow^{\text{deform}} f(z) + \mu\, g(z) = 0$$

    The singularity disappears!

Geometric Engineering

- The UV SCFTs are strongly-coupled ($1/g^2 = 0$) and are hard to define. But we can use geometry via **geometric engineering**!
- Consider **M-theory** (11d) on a geometry $\mathbb{R}^{1,4} \times M_6$. This M_6 is a 6d geometry called a **Calabi-Yau (CY)**, that "is compatible with SUSY".
- This CY should also be **singular**. These singularities can be removed in two ways:
  - **Resolution:** replace the singular point p by a sphere $S^2 \cong \mathbb{P}^1$.
  - **Deformation:** the CY is defined by polynomial equations

$$f(z) = 0 \to^{\text{deform}} f(z) + \mu\, g(z) = 0$$

    The singularity disappears!

- Then we can encode

$$\text{ECB} = \text{Resolutions}, \quad \text{HB} = \text{Deformations}$$

So *the singularity data is a dictionary for the QFT*!

Brane Junctions

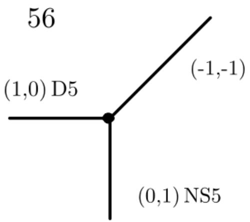
- We can also build 5d SCFTs from **Type IIB string theory** (10 d). The ingredients are:
  - **Dp branes**: Strings end on "hypersurfaces" of $p+1$ dimensions. We are interested on D5 and D7 branes.
  - **NS5 branes**: "magnetic dual" to the string, they have 5+1 dimensions.

  Charge: D5$=(1,0)$ and NS5$=(0,1)$ (can be "inverted" $(-1,0),(0,-1)$).

Brane Junctions

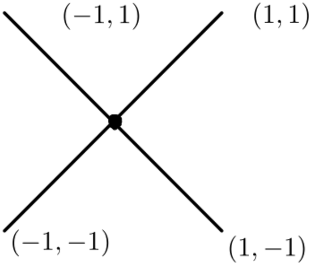
- We can also build 5d SCFTs from **Type IIB string theory** (10 d). The ingredients are:
  - **Dp branes**: Strings end on "hypersurfaces" of $p+1$ dimensions. We are interested on D5 and D7 branes.
  - **NS5 branes**: "magnetic dual" to the string, they have 5+1 dimensions.

  Charge: D5$=(1,0)$ and NS5$=(0,1)$ (can be "inverted" $(-1,0),(0,-1)$).
- We can create **junctions** between D5 and NS5 such that *the charge is conserved*. This preserves SUSY and results in a bound-brane of type $(-1,-1)$.



Brane Webs

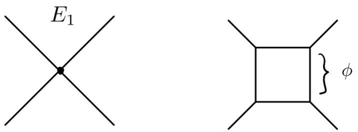
- We can consider more general bound states of (p, q) 5-branes that join at the same point such that charge is conserved/ it is SUSY.



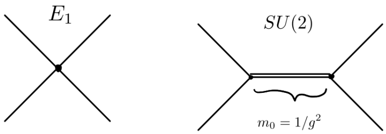
The SCFT lives at the 5d intersection of the branes and we make the 5-branes end on 7-branes (with the same (p, q) charge).

Extended Coulomb Branch in brane webs

- ECB moduli = breaking up the brane web into **irreducible supersymmetric 3-junctions**.
  We can open the web in two ways:
  - Keep the same asymptotic position of the external 5-branes= **Coulomb branch**.



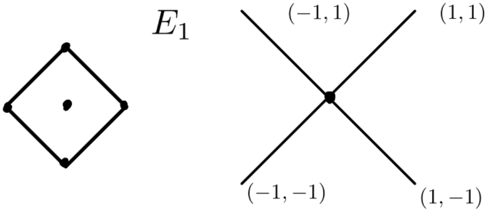
  - Alter the position of the external 5-branes= **mass deformations** (e.g. YM deformation).



Contents

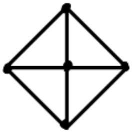
Toric Diagrams

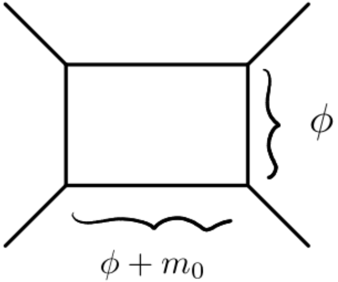
- If the Calabi-Yau M_6 is **toric** geometric engineering and brane webs are related! The brane web is dual (faces $\leftrightarrow$ vertices) to the **toric diagram** (encodes the geometry).



Resolutions in brane webs

- To fully resolve the toric singularity/fully open the ECB of a toric theory: we tile the toric diagram in **minimal triangles** $\leftrightarrow$ open the brane web in **irreducible 3-junctions**.





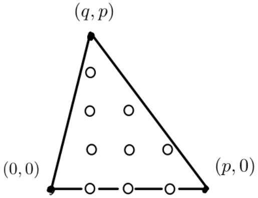
GTPs and The Tangram

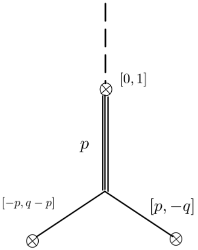
- We can make several (p, q) branes end on the same 7-brane $\rightarrow$ **Generalized Toric Polygon (GTP)**.

GTPs and The Tangram

- We can make several (p, q) branes end on the same 7-brane $\to$ **Generalized Toric Polygon (GTP)**.
- To fully resolve the ECB of a GTP, the fundamental piece of the tessellation is the **T-cones**:
  - Junction of a $(-p, q-p)$, $(p, -q)$ and p $(0, 1)$ branes.
  - Lattice triangle whose height and length are p.

Tiling the GTP polygon= ECB= solve the **Tangram**!





Thank You!